Numeric Response Questions

Monotonicity

Q.1 Function $f(x) = x^3 + 6x^2 + (9 + 2k)x + 1 \forall x \in R$ is strictly increasing function if $k > \lambda$ then find λ.

Q.2 The function $f(x) = \frac{a\sin x + 2\cos x}{\sin x + \cos x}$ is increasing for $x \in R$ when $a > \lambda$, then find λ,

Q.3 Let $\phi(x) = f(x) + f(2a - x)$ and $f''(x) > 0; \forall x \in [0, a]$. If $\phi(x)$ is decreasing in $[0, \lambda a]$ then find λ.

Q.4 Function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is decreasing when $x \in (a, b)$ then find value of $a + b$.

Q.5 The interval in which $2x^3 + 5$ increases less rapidly than $9x^2 - 12x$, is (a, b) then find a $+b$.

Q.6 If the function $f: R \to R$ given by $f(x) = x^3 + ax^2 + 5x + \sin 2x$ is invertible and $a \in (-3, \lambda)$ then find λ.

Q.7 If $f(x) = x^3 + 4x^2 + \lambda x + 1$ is monotonically decressing function of x in the largest possible interval $(-2, -2/3)$; then find λ

Q.8 If the complete set of values of x in which $f(x) = 2\log_a (x - 2) - x^2 + 4x + 1$ increases, is (a, b) then find $a + b$.

Q.9 If $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing $\forall x \in R$ and $k \in [\lambda, \infty)$, then find value of λ.

Q.10 If $f(x) = x - 2\sin x, 0 \leq x \leq 2\pi$ is increasing in the interval $[a\pi, bz]$, then find value of $a + b$.

ANSWER KEY

1. 1.50 **2.** 2.00 **3.** 1.00 **4.** 3.00 **5.** 3.00 **6.** 3.00 **7.** 4.00

8. 5.00 **9.** 3.00 **10.** 2.00

Hints & Solutions

1. $f'(x) = 3x^2 + 12x + (9 + 2k) > 0$
It is possible only when $B^2 - 4AC < 0$
$\Rightarrow 144 - 4.3\,(9 + 2k) < 0 \Rightarrow k > 3/2$

2. $f'(x) = \dfrac{a-2}{(\sin x + \cos x)^2}$
$\Rightarrow f'(x) > 0$ if $a > 2$

3. $\phi(x) = f(x) + f(2a - x)$
$\phi'(x) = f'(x) - f'(2a - x)$
In $[0, a]$
$f'(x) < f'(2a - x)$
$\Rightarrow \phi(x)$ is decreases on $[0, a]$

4. $f(x)$ is decreasing if
$f'(x) < 0$
$\Rightarrow 6x^2 - 18x + 12 < 0$
$\Rightarrow (x - 1)(x - 2) < 0$
$\Rightarrow 1 < x < 2$

5. $\dfrac{d}{dx}(2x^3 + 5) < \dfrac{d}{dx}(9x^2 - 12x)$
$6x^2 - 18x - 12 < 0$
$x^2 - 3x + 2 < 0$
$(1, 2)$

6. one-one and onto
$\dfrac{dy}{dx} = 3x^2 + 2ax + 5 + 2\cos 2x > 0$
$3x^2 + 2ax + 5 > -2\cos 2x$
$3x^2 + 2ax + 5 > 2$
$3x^2 + 2ax + 3 > 0$
$D < 0$
$4a^2 - 4 \times 3 \times 3 < 0$
$(a - 3)(a + 3) < 0$
$-3 < a < 3$

7. $f'(x) = 3x^2 + 8x + \lambda < 0$ in the largest interval $(-2, -2/3)$
$\therefore D = 64 - 12\lambda > 0$, $f'(-2) = 0$, $f'(-2/3) = 0$
$\therefore \lambda = 4$

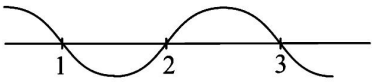
8. $f(x) = 2\log_e (x - 2) - x^2 + 4x + 1$

$$f'(x) = \frac{2}{x-2} - 2x + 4$$

$$= \frac{2}{x-2} - 2(x-2)$$

$$= \frac{2 - 2(x-2)^2}{x-2} = 2\frac{(1 + x - 2)(1 - x + 2)}{x-2}$$

$$= \frac{2(x-1)(3-x)}{x-2}$$



9. $f'(x) = 3kx^2 - 18x + 9$
$D \le 0 \Rightarrow 36 - 12k \le 0 \Rightarrow k \ge 3$

10. $f'(x) = 1 - 2\cos x \ge 0 \Rightarrow \cos x \le \dfrac{1}{2}$

$\therefore x \in \left[\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right]$ $\therefore a + b = \dfrac{1}{3} + \dfrac{5}{3} = 2$